2022 AB #2
(calculator-active)

(a)

$f(x)$ is the top curve and $g(x)$ is the bottom curve.

f and g intersect when $f(x) = g(x) \Rightarrow x = -2$ and $x = B = 0.7819751$.

$Area = \int_{-2}^{B} \left(f(x) - g(x) \right) dx = \boxed{3.6035}$

(b)

The vertical distance between f and g is $h(x) = f(x) - g(x)$.

$h'(-0.5) = f'(-0.5) - g'(-0.5) = -0.5999 < 0$.

h is $\boxed{\text{decreasing}}$ at $x = -0.5$ because $h'(x) < 0$ at $x = -0.5$.

(c)

The area of the cross sections are squares. So, $h(x) = f(x) - g(x)$ is the side of the square and the area of a cross section is $\left(h(x) \right)^2$.

$\therefore$ the volume of the solid is $\int_{-2}^{B} \left(h(x) \right)^2 dx = \boxed{5.340}$

(d)

From part (c), the area of a cross section is $A(x) = \left(h(x) \right)^2$.

$\frac{dx}{dt} = 7$. We need to find $\frac{dA}{dt}$ when $x = -0.5$.

$\frac{dA}{dt} = 2 \cdot h(x) \cdot h'(x) \cdot \frac{dx}{dt}$ (using careful use of the chain rule!)

$$\left. \frac{dA}{dt} \right|_{x=-0.5} = 2 \cdot h(-0.5) \cdot h'(-0.5) \cdot 7$$

$$= 2 \cdot \left(f(-0.5) - g(-0.5) \right) \cdot \left(f'(-0.5) - g'(-0.5) \right) \cdot 7$$

$$= 2 \cdot \left(f(-0.5) - g(-0.5) \right) \cdot \left(0.5999 \right) \cdot 7$$

$$= \boxed{-9.2718} \quad \text{or } -9.271 \text{ or } -9.272$$