## 2024 BC #5
**(no calculator)**

(a)

$h(x)=\int_0^x \sqrt{1+\left(f'(t)\right)^2}\,dt$

$h'(x)=\sqrt{1+\left(f'(x)\right)^2}$ by the Fundamental Theorem of Calculus

$h'(\pi)=\sqrt{1+\left(f'(\pi)\right)^2}=\boxed{\sqrt{1+(6)^2}}$ or $\sqrt{37}$

(b)

$\int_0^\pi \sqrt{1+\left(f'(x)\right)^2}\,dx$ is the arclength of $f$ from $x=0$ to $x=\pi$

(c)

Euler's Method: Two steps of equal size $\Rightarrow \Delta x=\dfrac{2\pi}{2}=\pi$

$f(0)=0$

$f(\pi)\approx f(0)+f'(0)\big(\pi-0\big)=0+5\pi=5\pi$

$f(2\pi)\approx f(\pi)+f'(\pi)\big(2\pi-\pi\big)=5\pi+6\pi=\boxed{11\pi}$

(d)

$\int (t+5)\cos\left(\dfrac{t}{4}\right)dt$ integration by parts: $u=t+5$ $v=4\sin\left(\dfrac{t}{4}\right)$

$du=dt$ $dv=\cos\left(\dfrac{t}{4}\right)$

$(t+5)\left(4\sin\left(\dfrac{t}{4}\right)\right)-\int 4\sin\left(\dfrac{t}{4}\right)dt$

$\boxed{(t+5)\left(4\sin\left(\dfrac{t}{4}\right)\right)+16\cos\left(\dfrac{t}{4}\right)+C}$