

2024 AB/BC #4
(no calculator)

(a)

$$g(-6) = \int_0^{-6} f(t) dt = -\int_{-6}^0 f(t) dt = \boxed{-12}$$

$$g(4) = \int_0^4 f(t) dt = \boxed{\frac{1}{2}(4)(2)} \text{ or } 4$$

$$g(6) = \int_0^6 f(t) dt = \boxed{4 - \frac{1}{2}(2)(1)} \text{ or } 3$$

(b)

g will have a critical point when $g'(x) = 0$ or undefined.

$$g'(x) = f(x) = 0 \text{ when } \boxed{x = 4}.$$

(c)

$$h(6) = \int_{-6}^6 f'(t) dt = f(6) - f(-6) = -1 - 0.5 = \boxed{-1.5}$$

$$h'(x) = f'(x)$$

$$h'(6) = f'(6) = \boxed{\frac{-1-2}{6-0}} \text{ or } -\frac{1}{2}$$

Note: This is the slope of the linear portion of the curve from $x = 0$ to $x = 7$.

$$h''(x) = f''(x)$$

$$h''(6) = f''(6) = \boxed{0}$$

Since curve is linear from $x = 0$ to $x = 7$ its slope, $f'(x)$, is constant so $f''(x) = 0$ there.