2024 AB/BC #1
(calculator-active)

(a)
Using the average rate of chage of C over the interval $3 \le t \le 7$,

$$C'(5) \approx \frac{C(7)-C(3)}{7-3} = \boxed{\frac{69-85}{7-3}\ \frac{\text{degrees Celsius}}{\text{minute}}} \text{ or } -4\ \frac{C^\circ}{\text{min}}$$

(b)
Using a left Riemann sum, $\int_0^{12} C(t)\,dt \approx (3-0)C(0)+(7-3)C(3)+(12-7)C(7)$

$$= \boxed{3(100) + 4(85) + 5(69)} \text{ or } 985$$

$\frac{1}{12}\int_0^{12} C(t)\,dt$ represents the average temperature of the coffee in the cup in degrees Celsius from time $t=0$ minutes to time $t=12$ minutes

(c)
$\int_{12}^{20} C'(t)\,dt = C(20)-C(12)$ or

$$C(20) = C(12) + \int_{12}^{20} C'(t)\,dt = 55 + \int_{12}^{20} C'(t)\,dt = \boxed{40.32918806\ C^\circ} \text{ or } 40.329\ C^\circ$$

(d)
We'll need the sign of C'' in order to determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate.

For $12 < t < 20$, $0.2455e^{0.01t}(100-t) > 0$ and $t^2 > 0$. Hence $C'' > 0$.

Therfore, on the given interval, the temperature of the coffee is changing at a increasing rate.