2024 AB #6
(no calculator)

(a)
$f(x)$ is the "top" curve, $g(x)$ is the "bottom" curve on $0 \le x \le 5$

The area of region R $= \int_0^2 \left(f(x) - g(x)\right) dx$

(b)
The volume of the solid with a given cross section is the integral of the area of a cross section on the given interval.

The area of the rectangular cross section $A(x)$ = (base of the rectangle)(height of the rectangle)

The base of the rectangle = $g(x) = x^2 - 2x$

The height of the rectangle is $\frac{1}{2}$(base) $= \frac{1}{2}g(x) = \frac{1}{2}(x^2 - 2x) = \frac{1}{2}x^2 - x$

So, $A(x) = \left(x^2 - 2x\right)\left(\frac{1}{2}x^2 - x\right) = \frac{1}{2}x^4 - x^3 - x^3 + 2x^2 = \frac{1}{2}x^4 - 2x^3 + 2x^2$

Volume of the solid $= \int_2^5 A(x)\,dx = \left(\frac{1}{10}x^5 - \frac{1}{2}x^4 + \frac{2}{3}x^3\right)\Bigg|_2^5$

$= \boxed{\left(\frac{1}{10}(5)^5 - \frac{1}{2}(5)^4 + \frac{2}{3}(5)^3\right) - \left(\frac{1}{10}(2)^5 - \frac{1}{2}(2)^4 + \frac{2}{3}(2)^3\right)}$

or $\frac{414}{5}$

(c)
When region S is rotated about the line $y = 20$ from $0 \le x \le 5,$ the cross sections are washers.

The area of the cross section $A(x) = \pi\left(\text{outer radius } R\right)^2 - \pi\left(\text{inner radius } r\right)^2$

$R = 20 - 0 = 20$, $r = 20 - g(x)$

So, $A(x) = \pi\left(20^2 - \left(20 - g(x)\right)^2\right)$

Volume of the solid = $\boxed{\pi \int_2^5 A(x)\,dx}$