2024 AB #5
(no calculator)

(a)

Tangent line: $y - y_1 = m(x - x_1)$ or $y = y_1 + \frac{dy}{dx}(x - x_1)$

$$\left.\frac{dy}{dx}\right|_{(2,4)} = \frac{-2(2)}{3+4(4)} = -\frac{4}{19}$$

Using the tangent line above at (2,4): When $x = 3$, $\boxed{y \approx 4 - \frac{4}{19}(3-2)}$ or $3\frac{15}{19}$

(b)

$\frac{dy}{dx} = 0$ only if $x = 0$ and $y \neq -\frac{4}{3}$.

If $x = 0$, then, because $y = 1$: $0^2 + 3(1) + 2(1)^2 = 5 \neq 48$

Therefore, $\boxed{\text{NO}}$, $y = 1$ cannot be tangent to the curve.

OR, ALTERNATIVELY:

If the horizontal line $y = 1$ is tangent to the curve, then $\frac{dy}{dx} = 0$ at $y = 1$.

So, $x^2 + 3(1) + 2(1)^2 = 48 \Rightarrow x^2 = 43 \Rightarrow x = \pm\sqrt{43}$

$\left.\frac{dy}{dx}\right|_{(\pm\sqrt{43},1)} \neq 0.$ So, NO, $y = 1$ cannot be tangent to the curve.

(c)

At $\left(\sqrt{48}, 0\right)$, $\frac{dy}{dx} = \frac{-2\sqrt{48}}{3+4(0)} = \frac{-2\sqrt{48}}{3}$ which is a Real number.

Therefore, the tangent line is not vertical at $\left(\sqrt{48}, 0\right)$ since the slope is not undefined there.

(d)

For $y^3 + 2xy = 24$, at $(4,2)$ $\frac{dy}{dt} = -2$. We need $\frac{dx}{dt}$.

$$3y^2\frac{dy}{dt} + 2x\frac{dy}{dt} + y(2)\frac{dx}{dt} = 0$$

At $(4,2)$: $3(2)^2(-2) + 2(4)(-2) + 2(2)\frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = \boxed{10}$