

2023 BC #5
(no calculator)

(a)

$$\begin{aligned} \text{Area} &= \int_0^3 (f(x) - g(x)) dx = \int_0^3 f(x) dx - \int_0^3 g(x) dx \\ &= \boxed{10 - 12(\ln 6 - \ln 3)} \\ &\text{or } 10 - 12 \ln 2 \end{aligned}$$

$$\begin{aligned} \text{Note: } \int_0^3 g(x) dx &= \int_0^3 \frac{12}{3+x} dx \\ &= 12 \ln|3+x| \Big|_0^3 \\ &= 12(\ln 6 - \ln 3) \\ &= 12 \ln \frac{6}{3} = 12 \ln 2 \end{aligned}$$

(b)

$$\begin{aligned} \int_0^\infty (g(x))^2 dx &= \lim_{b \rightarrow \infty} \int_0^b \left(\frac{12}{3+x} \right)^2 dx \\ &= \lim_{b \rightarrow \infty} \int_0^b 144(3+x)^{-2} dx \\ &= \lim_{b \rightarrow \infty} -144(3+x)^{-1} \Big|_0^b \\ &= -144 \lim_{b \rightarrow \infty} \left(\frac{1}{3+b} - \frac{1}{3+0} \right) \\ &= \boxed{-144 \left(0 - \frac{1}{3} \right)} \text{ or } \frac{144}{3} \text{ or } 48 \end{aligned}$$

(c)

$$h(x) = x \cdot f'(x)$$

$$\int_0^3 h(x) dx = \int_0^3 x \cdot f'(x) dx$$

$$= x \cdot f(x) \Big|_0^3 - \int_0^3 f(x) dx$$

$$= 3f(3) - 0f(0) - 10$$

$$= \boxed{3(2) - 0 - 10} \text{ or } -4$$

Using integration by parts: $u = x$ $v = f(x)$

$$du = dx \quad dv = f'(x) dx$$