

2023 AB/BC #1
(calculator-active)

(a)

$\int_{60}^{135} f(t) dt$ is the number of gallons of gasoline pumped into the gas tank from $t = 60$ seconds until $t = 135$ seconds from when the pumping began.

Using a right Riemann sum,

$$\int_{60}^{135} f(t) dt \approx (90 - 60)f(90) + (120 - 90)f(120) + (135 - 120)f(135)$$

$$= \boxed{(90 - 60)(0.15) + (120 - 90)(0.1) + (135 - 120)(0.05) \text{ gallons}} = 8.25 \text{ gallons}$$

(b)

Since f is differentiable on the given interval, then it is continuous on that interval.

Hence f is differentiable and continuous on the interval $[60, 120]$.

So by the Mean Value Theorem there must exist a time $t = c$, for $60 < c < 120$, such that

$$f'(c) = \frac{f(120) - f(60)}{120 - 60} = \frac{0.1 - 0.1}{60} = 0.$$

(c)

The average rate of flow on $[0, 150] = \frac{1}{150 - 0} \int_0^{150} g(t) dt = \boxed{0.0959966962 \frac{\text{gal}}{\text{sec}}}$ or 0.095 or 0.096

(d)

$$g'(140) = \boxed{-0.004908888} \text{ or } -0.004 \text{ or } -0.005$$

The rate of flow of the gasoline into the tank is decreasing at $0.00490888 \frac{\text{gal}}{\text{sec}}$

140 seconds after pumping began.