2023 AB #6
(no calculator)

(a)

$6xy = 2 + y^3$

$6x\dfrac{dy}{dx} + y \bullet 6 = 3y^2\dfrac{dy}{dx} \rightarrow \left(6x - 3y^2\right)\dfrac{dy}{dx} = -6y$

$\dfrac{dy}{dx} = \dfrac{-6y}{6x - 3y^2} = \dfrac{-6y}{6x - 3y^2}\left(\dfrac{\frac{-1}{3}}{\frac{-1}{3}}\right) = \dfrac{2y}{y^2 - 2x}$

(b)

The tangent line is horizontal if it's slope at the point of tangency, (x, y), is zero $\rightarrow \left.\dfrac{dy}{dx}\right|_{(x,y)} = 0$

and (x, y) exits. Now, $\dfrac{dy}{dx} = 0$ when $2y = 0 \rightarrow y = 0$

Finding x when $y = 0$ using $6xy = 2 + y^3 \rightarrow 6x(0) = 0$ and $2 + (0)^3 = 2 \rightarrow$ But $0 \neq 2$

So no point (x, y) exists when $y = 0$ and $\left.\dfrac{dy}{dx}\right|_{(x,y)} = 0$ and no horizontal tangent line exists.

(c)

The tangent line is vertical when the slope of the tangent line is undefined at the point of tangency, $(x, y) \rightarrow$

$\left.\dfrac{dy}{dx}\right|_{(x,y)}$ is undefined and (x, y) exists.

$\dfrac{dy}{dx}$ is undefined when $y^2 - 2x = 0 \rightarrow 2x = y^2 \rightarrow x = \dfrac{y^2}{2}$

Finding y when $x = \dfrac{y^2}{2}$ using $6xy = 2 + y^3$:

$6\left(\dfrac{y^2}{2}\right)y = 2 + y^3 \rightarrow 3y^3 = 2 + y^3 \rightarrow y^3 = 1 \rightarrow y = 1$

and since $x = \dfrac{y^2}{2} \rightarrow x = \dfrac{1}{2}$

So since $6\left(\dfrac{1}{2}\right)(1) = 2 + (1)^3$ the tangent line is vertical at $\boxed{\left(\dfrac{1}{2}, 1\right)}$ because the slope of

the tangent line is undefined there and the point exists.

(d)

At the point $\left(\dfrac{1}{2}, -2\right)$, $\dfrac{dx}{dt} = \dfrac{2}{3}$. We must find $\dfrac{dy}{dt}$ there. Since $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} \rightarrow \dfrac{dy}{dt} = \dfrac{dy}{dx} \bullet \dfrac{dx}{dt}$

$\left.\dfrac{dy}{dx}\right|_{(1/2,-2)} = \dfrac{2(-2)}{(-2)^2 - 2\left(\frac{1}{2}\right)} = \dfrac{-4}{4 - 1} = -\dfrac{4}{3}$ So $\dfrac{dy}{dt} = \dfrac{dy}{dx} \bullet \dfrac{dx}{dt} = \boxed{-\dfrac{4}{3} \bullet \dfrac{2}{3}}$ or $-\dfrac{8}{9}$