2018 BC #5
(no calculator)

(a)

Area inside of $r = 4$ and outside of $r = 3 + 2\cos\theta = \boxed{\frac{1}{2}\int_{\pi/3}^{5\pi/3}\left[(4)^2 - (3+2\cos\theta)^2\right]d\theta}$

(b)

slope $m = \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$ and $r = 3 + 2\cos\theta$

$y = r\sin\theta = (3+2\cos\theta)\sin\theta = 3\sin\theta + 2\cos\theta\sin\theta = 3\sin\theta + \sin 2\theta$

$\dfrac{dy}{d\theta} = 3\cos\theta + 2\cos 2\theta \quad \Rightarrow \quad \left.\dfrac{dy}{d\theta}\right|_{\theta=\pi/2} = 3\cos\dfrac{\pi}{2} + 2\cos\pi = -2$

$x = r\cos\theta = (3+2\cos\theta)\cos\theta = 3\cos\theta + 2\cos^2\theta$

$\dfrac{dx}{d\theta} = -3\sin\theta - 4\cos\theta\sin\theta \quad \Rightarrow \quad \left.\dfrac{dx}{d\theta}\right|_{\theta=\pi/2} = -3\sin\dfrac{\pi}{2} - 4\cos\dfrac{\pi}{2}\sin\dfrac{\pi}{2} = -3$

slope $m\big|_{\theta=\pi/2} = \left.\dfrac{dy/d\theta}{dx/d\theta}\right|_{\theta=\pi/2} = \dfrac{-2}{-3} = \boxed{\dfrac{2}{3}}$

(c)

$r = 3 + 2\cos\theta$ and $\dfrac{dr}{dt} = 3$. Find $\dfrac{d\theta}{dt}$ when $\theta = \dfrac{\pi}{3}$.

$\dfrac{dr}{dt} = -2\sin\theta\left(\dfrac{d\theta}{dt}\right) \quad \Rightarrow \quad 3 = -2\sin\dfrac{\pi}{3}\left(\dfrac{d\theta}{dt}\right)$

$\Rightarrow \quad 3 = -2\left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{d\theta}{dt}\right) \quad \Rightarrow \quad \dfrac{d\theta}{dt} = \boxed{-\dfrac{3}{\sqrt{3}}\dfrac{\text{radians}}{\text{second}}}$ or $-\sqrt{3}\dfrac{\text{radians}}{\text{second}}$