2018 BC #2
(calculator-active)

(a)

Using the calculator and the given expression for $p(h)$:

$p'(25) \approx \boxed{-1.179064048}$ or -1.179

This means that at a depth of 25 meters, the density of the plankton is decreasing at a rate of $1.179064048 \dfrac{\frac{\text{millions of cells}}{\text{cubic meter}}}{\text{meter}}$.

(b)

Think of finding the number of cells as a Riemann sum and use units:

$$\text{millions of cells} = \frac{\text{millions of cells}}{\text{m}^3} \bullet \text{m}^2 \bullet \text{m} = \text{density} \bullet \text{area} \bullet \Delta h$$

So, the number of millions of cells between $h = 0$ and $h = 30$ is

$$\int_0^{30} p(h) \bullet 3 \bullet dh \approx 1675.414936 = a$$

To the nearest million: $\boxed{\text{1675 million cells}}$

(c)

Millions of plankton cells in the entire column:

It is given that $0 \le f(h) \le u(h)$ for $h \ge 30$ and $\int_{30}^{K} u(h)\,dh = 105$.

$$\begin{aligned}
\text{Millions of cells} &= \int_0^{30} p(h) \bullet 3 \bullet dh + \int_{30}^{K} f(h) \bullet 3 \bullet dh \\
&= a + \int_{30}^{K} f(h) \bullet 3 \bullet dh \\
&\le a + \int_{30}^{K} u(h) \bullet 3 \bullet dh \\
&\le a + \int_{30}^{\infty} u(h) \bullet 3 \bullet dh \\
&= a + 3(105) \\
&= 1990.415 \\
&\le 2000
\end{aligned}$$

(d)

$$\text{Total distance traveled over } [0,1] = \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

$\approx \boxed{\text{757.4558623 meters}}$ or 757.456 m