2018 AB/BC #4
(no calculator)

H is twice differentiable $\Rightarrow$ H and H' are continuous.

(a)

$$H'(6) \approx \frac{H(7)-H(5)}{7-5} = \boxed{\frac{11-6}{7-5}} \text{ or } \frac{5}{2}$$

This means that the height of the tree is growing at a rate of $\frac{5}{2}\frac{\text{meters}}{\text{year}}$ at $t = 6$ years.

(b)

We see that $\frac{H(5)-H(3)}{5-3} = \frac{6-2}{5-3} = 2$ and we know that since H is differentiable on $[2,10]$ then H is continuous on $[2,10]$. So we know that H is both continuous and differentiable on $[3,5]$. So, by the Mean Value Theorem, we know that there must be at least one t on $(3,5)$ such that $H'(t) = 2$, hence, one such value exists on $(2,10)$.

(c)

$$H_{avg} = \frac{1}{10-2}\int_2^{10} H(t)\,dt$$

Using a trapezoidal sum to approximate the integral:

$$H_{avg} \approx \boxed{\frac{1}{8}\left[\frac{1}{2}(1)(1.5+2)+\frac{1}{2}(2)(2+6)+\frac{1}{2}(2)(6+11)+\frac{1}{2}(3)(11+15)\right]} \text{ or } \frac{263}{32} \text{ meters}$$

(d)

When $G(x) = \frac{100x}{1+x} = 50 \Rightarrow x = 1$ and $\frac{dx}{dt} = .03$

$$\frac{dG}{dt} = \frac{100\left(\frac{dx}{dt}\right)(1+x) - 100x\left(\frac{dx}{dt}\right)}{(1+x)^2} = \frac{100}{(1+x)^2}\left(\frac{dx}{dt}\right)$$

When $G(x) = 50 \Rightarrow \frac{dG}{dt} = \boxed{\frac{100}{(1+1)^2}(.03)\frac{\text{meters}}{\text{year}}} \text{ or } \frac{3}{4}$