2018 AB/BC #3
(no calculator)

The graph of $g = f'$ is given.

(a)

By the Fundamental Theorem of Calculus, $\int_{-5}^{1} f'(x)\,dx = f(1) - f(-5)$

$$f(-5) = f(1) - \int_{-5}^{1} f'(x)\,dx = 3 - \int_{-5}^{1} g(x)\,dx$$

$$= \boxed{3 - \left[-3(3) - \frac{1}{2}(1)(3) + \frac{1}{2}(1)(2)\right]} \text{ or } \frac{25}{2}$$

(b)

$$\int_{1}^{6} g(x)\,dx = \int_{1}^{3} g(x)\,dx + \int_{3}^{6} g(x)\,dx = 2(2) + \int_{3}^{6} 2(x-4)^2\,dx$$

$$= 4 + \left[\frac{2}{3}(x-4)^3\right]_3^6 = \boxed{4 + \left(\frac{2}{3}(8) - \frac{2}{3}(-1)\right)} \text{ or } 10$$

(c)

f is increasing when f' or g is positive and this occurs when $0 < x < 4$ and $4 < x < 6$.

f is concave up when f'' or g' is positive or when f' or g is increasing and this occurs when $-2 < x < -1$, $0 < x < 1$, and $4 < x < 6$.

f is both increasing and concave up when $\boxed{0 < x < 1 \text{ or } 4 < x < 6}$.

(d)

f has an inflection point when f'' or g' changes signs or when f' or g changes from increasing to decreasing or decreasing to increasing and this occurs at $\boxed{x = 4}$.