

2018 RELEASED FREE RESPONSE SOLUTIONS – MR. CALCULUS

2018 AB/BC #1
(calculator-active)

(a)

Since $r(t)$ is the rate that people enter the line, then the number of people

that enter the line from $0 \leq t \leq 300$ would be $\int_0^{300} r(t) dt = \boxed{270 \text{ people}}$

(b)

The number of people in line at a time t is the number of people in line at $t = 0$ plus the number

of people that enter the line minus the number of people that exit the line: $P(t) = P(0) + \int_0^t r(x) dx - 0.7t$

So, $P(300) = 20 + \int_0^{300} r(x) dx - 0.7(300) = \boxed{20 + 270 - 0.7(300) \text{ people}}$ or 80 people

(c)

There are no people in line when $P(t) = 20 + 270 - 0.7t = 0$

$\Rightarrow t \approx \boxed{414.2857143 \text{ seconds}}$ or 414.286

(d)

On $[0, 300]$, $P(t)$ is a minimum when $t = 0, t = 300$, or when $P'(t) = 0$.

Solving on the calculator: $P'(t) = r(t) - 0.7 = 0 \Rightarrow t = 33.013298 \text{ sec} = a$

$t = 166.57472 \text{ sec} = b$

$$P(0) = 20$$

$$P(300) = 80$$

$$P(a) = 20 + \int_0^a r(x) dx - 0.7a \approx 3.803436931$$

$$P(b) = 20 + \int_0^b r(x) dx - 0.7b \approx 158.07014$$

The number of people in line is a minimum on the interval when $t = \boxed{a \text{ sec}}$

At this time, there are $\boxed{4}$ people in line.