2018 AB #5
(no calculator)

(a)
Average rate of change of f on $[0,\pi]$

$$= \frac{1}{\pi - 0}\int_0^\pi f'(x)\,dx = \frac{f(\pi) - f(0)}{\pi - 0} = \frac{e^\pi \cos\pi - e^0 \cos 0}{\pi} = \boxed{\frac{-e^\pi - 1}{\pi}}$$

(b)
$f'(x) = e^x(-\sin x) + e^x(\cos x)$

$$m\Big|_{x=3\pi/2} = f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2}\left(-\sin\frac{3\pi}{2}\right) + e^{3\pi/2}\left(\cos\frac{3\pi}{2}\right) = \boxed{e^{3\pi/2}}$$

(c)
The absolute minimum of f on $[0,2\pi]$ will occur at $x=0$, $x=2\pi$, or where $f'(x)=0$.

$$f'(x) = e^x\left(-\sin x + \cos x\right) = 0 \;\Rightarrow\; \sin x = \cos x \;\Rightarrow\; x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$f(0) = e^0 \cos 0 = 1$

$f(2\pi) = e^{2\pi}\cos 2\pi = e^{2\pi}$

$$f\left(\frac{\pi}{4}\right) = e^{\pi/4}\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}e^{\pi/4}$$

$$f\left(\frac{5\pi}{4}\right) = e^{5\pi/4}\cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}e^{5\pi/4}$$

Of these candidates, the absolute minimum of f on $[0,2\pi]$ is $\boxed{-\frac{\sqrt{2}}{2}e^{5\pi/4}}$.

(d)
We know that the given function f is continuous and differentiable at $x=\frac{\pi}{2}$ so $\lim\limits_{x\to\pi/2} f(x) = f\left(\frac{\pi}{2}\right) = 0.$

It was given that g is differentiable and, therefore, continuous at $x=\frac{\pi}{2}$ so $\lim\limits_{x\to\pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0.$

So we can use L'Hospital's Rule to find $\lim\limits_{x\to\pi/2}\frac{f(x)}{g(x)}$.

$$\lim_{x\to\pi/2} f'(x) = f'\left(\frac{\pi}{2}\right) = e^{\pi/2}\left(-\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right) = -e^{\pi/2}$$

From the graph of g': $\lim\limits_{x\to\pi/2} g'(x) = 2$

So, using L'Hospital's Rule: $\lim\limits_{x\to\pi/2}\frac{f(x)}{g(x)} = \lim\limits_{x\to\pi/2}\frac{f'(x)}{g'(x)} = \boxed{\frac{-e^{\pi/2}}{2}}$