2017 BC #5
(no calculator)

(a)

$$f(x)=3\left(2x^2-7x+5\right)^{-1} \quad \Rightarrow \quad f'(x)=-3\left(2x^2-7x+5\right)^{-2}\left(4x-7\right)=\frac{-3\left(4x-7\right)}{\left(2x^2-7x+5\right)^2}$$

$$m\Big|_{x=3}=f'(3)=\frac{-3(12-7)}{\left(18-21+5\right)^2} \quad \text{or} \quad \frac{-15}{4}$$

(b)

$$f'(x)=\frac{-3(4x-7)}{\left[(2x-5)(x-1)\right]^2}$$

$f'(x)=0 \;\Rightarrow\; x=\dfrac{7}{4}$ Note: $f'(x)$ is undefined at $x=1$ and $x=\dfrac{5}{2}$ but these are not in $\left(1,2.5\right)$.

$f'(x)>0$ for $\left(1,\dfrac{7}{4}\right)$ and $f'(x)<0$ for $\left(\dfrac{7}{4},2.5\right)$.

In $\left(1,2.5\right)$ f has a **relative maximum at $x=\dfrac{7}{4}$** because $f'(x)$ changes from positive to negative there.

(c)

$$\int_5^{\infty} f(x)\,dx=\lim_{b\to\infty}\int_5^b\left(\frac{2}{2x-5}-\frac{1}{x-1}\right)dx=\lim_{b\to\infty}\Big[\ln(2x-5)-\ln(x-1)\Big]_5^b=\lim_{b\to\infty}\left[\ln\left(\frac{2x-5}{x-1}\right)\right]_5^b$$

$$=\lim_{b\to\infty}\left[\ln\left(\frac{2b-5}{b-1}\right)-\ln\left(\frac{5}{4}\right)\right]=\ln 2-\ln\frac{5}{4} \quad \text{or} \quad \ln\left(\frac{2}{5/4}\right) \quad \text{or} \quad \ln\frac{8}{5}$$

(d)

The series converges.

On $[5,\infty)$, $f(x)$ is continuous, positive, and, since $f'(x)<0$ on the interval, decreasing.

Since the integral in part (c) converges, then by the Integral Test, the series given converges.

OR

Since all terms of the series are positive, you could use the Limit Comparison Test with $\displaystyle\sum_{n=5}^{\infty}\frac{1}{n^2}$ which converges because it is a p-series where $p>1$. Now, $\displaystyle\lim_{n\to\infty}\frac{\left(\dfrac{3}{2n^2-7n+5}\right)}{\left(\dfrac{1}{n^2}\right)}=\frac{3}{2}>0$ and finite.

Therefore the series given converges by the Limit Comparison Test.