

2017 AB/BC #4
(no calculator)

(a)

$$m|_{t=0} = H'(0) = -\frac{1}{4}(91-27) = -16 \Rightarrow H-91 = -16(t-0)$$

$$H(3) \approx 91 - 16(3) \text{ } ^\circ\text{C} \text{ or } 43 \text{ } ^\circ\text{C}$$

(b)

$$\frac{d^2H}{dt^2} = -\frac{1}{4}\left(\frac{dH}{dt}\right) = -\frac{1}{4}\left[-\frac{1}{4}(H-27)\right] = \frac{1}{16}(H-27)$$

It was given in the problem that $H > 27$ for $t > 0$. So, for all $t > 0$, $\frac{d^2H}{dt^2} > 0$ and $H(t)$ is concave up.

\therefore my answer for $H(3)$ in part (a) is an **underestimate**.

(c)

$$\frac{dG}{(G-27)^{2/3}} = -dt$$

$$\int (G-27)^{-2/3} dG = \int -dt$$

$$3(G-27)^{1/3} = -t + C \quad \text{Since } G(0) = 91, 3(91-27)^{1/3} = 0 + C \Rightarrow C = 12$$

$$\text{So } 3(G-27)^{1/3} = -t + 12$$

$$(G-27)^{1/3} = \frac{12-t}{3}$$

$$G-27 = \left(\frac{12-t}{3}\right)^3 \Rightarrow G(t) = 27 + \left(\frac{12-t}{3}\right)^3$$

$$G(3) = 27 + \left(\frac{12-3}{3}\right)^3 \text{ or } 54$$