2017 AB/BC #4
(no calculator)

(a)

$H(0) = 91$

$m\big|_{t=3} = H'(0) = -\frac{1}{4}(91-27) = -16 \implies H - 91 = -16(t-0)$

$H(3) \approx 91 - 16(3)$ °C or 43 °C

(b)

$\frac{d^2H}{dt^2} = -\frac{1}{4}\left(\frac{dH}{dt}\right) = -\frac{1}{4}\left[-\frac{1}{4}(H-27)\right] = \frac{1}{16}(H-27)$

It was given in the problem that $H > 27$ for $t > 0$. So, for all $t > 0$, $\frac{d^2H}{dt^2} > 0$ and $H(t)$ is concave up.

$\therefore$ my answer for $H(3)$ in part (a) is an **underestimate**.

(c)

$\frac{dG}{(G-27)^{2/3}} = -dt$

$\int (G-27)^{-2/3}\, dG = \int -dt$

$3(G-27)^{1/3} = -t + C$ Since $G(0) = 91$, $3(91-27)^{1/3} = 0 + C \implies C = 12$

So $3(G-27)^{1/3} = -t + 12$

$(G-27)^{1/3} = \frac{12-t}{3}$

$G - 27 = \left(\frac{12-t}{3}\right)^3 \implies G(t) = 27 + \left(\frac{12-t}{3}\right)^3$

$G(3) = 27 + \left(\frac{12-3}{3}\right)^3$ or 54 °C