2017 AB/BC #3
(no calculator)

f is differentiable so f is continuous on $\left[-6,5\right]$ and $f(-2)=7$.

(a) $\int_{-2}^{x} f'(t)\,dt = f(x)-f(-2)$ by the FTC $f(-6)=f(-2)+\int_{-2}^{-6} f'(t)\,dt = 7-\int_{-6}^{-2} f'(t)\,dt = 7-\frac{1}{2}(4)(2)$ or 3 $f(5)=f(-2)+\int_{-2}^{5} f'(t)\,dt = 7-\frac{1}{2}(\pi)(2)^2+\frac{1}{2}(3)(2)$ or $10-2\pi$
(b) Since $f'(x)>0$ when $-6\le x<-2$ and $2<x<5$, f is increasing when $-6\le x\le -2$ and $2\le x\le 5$. Note: The inclusion of the endpoints (and not) are usually accepted.
(c) The absolute minimum value of f on $\left[-6,5\right]$ will occur at the endpoints of the interval or at critical points of f on the interval. The values at the endpoints were found in part (a): $f(-6)=3$ $f(5)=10-2\pi$ Critical points occur when $f'(x)=0$, at $x=-2,\ 2,\ 5$: $f(-2)=7$ (this was given) $f(2)=f(-2)+\int_{-2}^{2} f'(t)\,dt = 7-2\pi$ The smallest value of these is the absolute minimum value on the interval and it is $7-2\pi$.
(d) $f''(-5)=\dfrac{f'(-2)-f'(-6)}{-2-(-6)}=\dfrac{0-2}{4}$ or $-\dfrac{1}{2}$ $f''(3)$ **does not exist** because the slope of $f'(x)$ at $x=3$ does not exist since $\lim\limits_{x\to 3^-} f''(x)=2\ne -1=\lim\limits_{x\to 3^+} f''(x)$.