

2017 AB/BC #3
(no calculator)

f is differentiable so f is continuous on $[-6, 5]$ and $f(-2) = 7$.

(a)

$$\int_{-2}^x f'(t) dt = f(x) - f(-2) \text{ by the FTC}$$

$$f(-6) = f(-2) + \int_{-2}^{-6} f'(t) dt = 7 - \int_{-6}^{-2} f'(t) dt = 7 - \frac{1}{2}(4)(2) \text{ or } 3$$

$$f(5) = f(-2) + \int_{-2}^5 f'(t) dt = 7 - \frac{1}{2}(\pi)(2)^2 + \frac{1}{2}(3)(2) \text{ or } 10 - 2\pi$$

(b)

f is increasing when $-6 \leq x \leq -2$ and $2 \leq x \leq 5$

since f' is non-negative on these intervals.

(c)

The absolute minimum value of f on $[-6, 5]$ will occur at the endpoints of the interval or at critical points of f on the interval.

The values at the endpoints were found in part (a):

$$f(-6) = 3$$

$$f(5) = 10 - 2\pi$$

Critical points occur when $f'(x) = 0$, at $x = -2, 2, 5$:

$$f(-2) = 7 \text{ (this was given)}$$

$$f(2) = f(-2) + \int_{-2}^2 f'(t) dt = 7 - 2\pi$$

The smallest value of these is the absolute minimum value on the interval and it is $7 - 2\pi$.

(d)

$$f''(-5) = \frac{f'(-2) - f'(-6)}{-2 - (-6)} = \frac{0 - 2}{4} \text{ or } -\frac{1}{2}$$

$f''(3)$ **does not exist** because the slope of $f'(x)$ at $x = 3$ does not exist

since $\lim_{x \rightarrow 3^-} f''(x) = 2 \neq -1 = \lim_{x \rightarrow 3^+} f''(x)$.