2017 AB/BC #3
(no calculator)

f is differentiable so f is continuous on $\left[-6,5\right]$ and $f(-2)=7$.

(a)

$\int_{-2}^{x} f'(t)\,dt = f(x) - f(-2)$ by the FTC

$f(-6) = f(-2) + \int_{-2}^{-6} f'(t)\,dt = 7 - \int_{-6}^{-2} f'(t)\,dt = 7 - \frac{1}{2}(4)(2)$ or 3

$f(5) = f(-2) + \int_{-2}^{5} f'(t)\,dt = 7 - \frac{1}{2}(\pi)(2)^2 + \frac{1}{2}(3)(2)$ or $10 - 2\pi$

(b)

Since $f'(x) > 0$ when $-6 \le x < -2$ and $2 < x < 5$,

f is increasing when $-6 \le x \le -2$ and $2 \le x \le 5$.

Note: The inclusion of the endpoints (and not) are usually accepted.

(c)

The absolute minimum value of f on $\left[-6,5\right]$ will occur at the endpoints of the interval or at critical points of f on the interval.

The values at the endpoints were found in part (a):

$f(-6) = 3$

$f(5) = 10 - 2\pi$

Critical points occur when $f'(x) = 0$, at $x = -2,\ 2,\ 5$:

$f(-2) = 7$ (this was given)

$f(2) = f(-2) + \int_{-2}^{2} f'(t)\,dt = 7 - 2\pi$

The smallest value of these is the absolute minimum value on the interval and it is $7 - 2\pi$.

(d)

$f''(-5) = \frac{f'(-2) - f'(-6)}{-2-(-6)} = \frac{0-2}{4}$ or $-\frac{1}{2}$

$\lim\limits_{x\to 3^-} \frac{f'(x) - f'(3)}{x-3} = 2$ and $\lim\limits_{x\to 3^+} \frac{f'(x) - f'(3)}{x-3} = -1$.

$f''(3)$ **does not exist** because the slope of $f'(x)$ at $x = 3$ does not exist because

$\lim\limits_{x\to 3^-} \frac{f'(x) - f'(3)}{x-3} \ne \lim\limits_{x\to 3^+} \frac{f'(x) - f'(3)}{x-3}$