2017 AB #6
(no calculator)

g is given as a differentiable function so g is a continuous function.

(a)

$f'(x) = -2\sin(2x) + (\cos x)e^{\sin x}$

$m\big|_{x=\pi} = f'(\pi) = -2\sin(2\pi) + (\cos\pi)e^{\sin\pi} = 0 - 1(e^0)$ or -1

(b)

Using the chain rule:

$k'(x) = h'(f(x))f'(x)$

$k'(\pi) = h'(f(\pi))f'(\pi)$ $\qquad f(\pi) = \cos(2\pi) + e^{\sin\pi} = 1 + e^0 = 2$

$= h'(2)(-1)$ $\qquad h'(2) = \dfrac{h(3)-h(0)}{3-0} = \dfrac{-1-0}{3} = -\dfrac{1}{3}$

$= -\dfrac{1}{3}(-1)$ or $\dfrac{1}{3}$

(c)

Using the product rule:

$m'(x) = g(-2x)h'(x) + h(x)(-2g'(-2x))$

$m'(2) = g(-4)h'(2) - 2h(2)g'(-4)$ $\qquad$ Since $\dfrac{h(2)-h(0)}{2-0} = -\dfrac{1}{3}$, then $h(2) = -\dfrac{2}{3}$.

$= 5\left(-\dfrac{1}{3}\right) - 2\left(-\dfrac{2}{3}\right)(-1)$ or -3

(d)

Yes

Since g is differentiable for all x, then it is continuous for all x. Hence it is certainly continuous on $[-5,-3]$ and differentiable on $(-5,-3)$ thus satisfying the conditions of the Mean Value Theorem.

Therefore there exists a c in $(-5,-3)$ such that $g'(c) = \dfrac{g(-3)-g(-5)}{-3-(-5)} = \dfrac{2-10}{2} = -4$

Since there exists a c in $(-5,-3)$, then clearly there exists a c in $[-5,-3]$.