2017 AB #5
(no calculator)

(a)

$v_P(t) = x'_P(t) = \dfrac{2t-2}{t^2-2t+10} = 0$

$v_P(t) = 0 \Rightarrow t = 1$ and $v_P(t) < 0$ on $[0,1)$ and $v_P(t) > 0$ on $(1,8]$.

On $[0,8]$, the particle moves to the left when $v_P(t) < 0$ and this occurs on $[0,1)$.

(b)

$v_Q(t) = (t-5)(t-3) = 0$ when $t = 3,$ and $t = 5.$

$v_Q(t) > 0$ on $[0,3)$ and $(5,8]$ and $v_Q(t) <$ on $(3,5)$

This tells us that particle Q moves to the right on $[0,3)$ and $(5,8]$ and left on $(3,5)$.

So the particles move the same direction on $(1,3)$ and $(5,8]$

since $v_P(t)$ and $v_Q(t)$ have the same signs on these intervals.

(c)

$a_Q(t) = v'_Q(t) = 2t-8 \quad \Rightarrow \quad a_Q(2) = 2(2)-8$ or -4

At $t = 2$ the speed of particle Q is **decreasing** because $a_Q(2)$ and $v_Q(2)$ have different signs.

Note: $a_Q(2) < 0$ and from part (b), $v_Q(2) > 0$ which means that particle Q is moving to the right at a decreasing rate. Hence at $t = 2$ the particle is slowing down or the speed of the particle is decreasing .

(d)

The position of particle Q first changes direction when $t = 3,$ so

$$x_Q(3) = x_Q(0) + \int_0^3 v_Q(t)\,dt$$

$$= 5 + \left[\frac{1}{3}t^3 - 4t^2 + 15t\right]_0^3$$

$$= 5 + \left(9 - 36 + 45\right) \text{ or } 23$$